

Midterm I, Oct. 14th 2014

Note: The first midterm will cover the following topics. 1. stereographic projection (definition and the way to find a projecting point) 2. calculate square root for a given complex number 3. triangle inequality 4. differentiation of a holomorphic function. (definition, Cauchy-Riemann equation, method to calculate a derivative for a given function, find a harmonic conjugate for a given real part) 5. Maximal modulus theorem (proof is not required, just need know how to use it) 6. write a rational function into a sum of partial fractions 7. linear transformation (cross ratio, the way to find a center of a circle decided by three points, how to decide if four points are on a same circle, symmetric points, reflection with respect to a circle, determine a linear transformation which can realize some transformations between circles)

Here are some sample problems.

Problem 1: write the rational function

$$R(z) = \frac{3z + i}{z(z + 1)(z + 2)}$$

into sum of partial fractions.

Problem 2: Determine the holomorphic functions f and g so that $\operatorname{Re} f = x^2 - y^2 - 2y$, $\operatorname{Im} g = 2xy + y$. calculate the derivatives of f and g .

Problem 3: It has been shown that a linear transformation sends a circle to a circle.

- (a) Find a circle C which can be mapped to the circle $|z - 2| = 1$ by the linear transformation $Sz = 2z/(z + i)$;
- (b) What is the image of $|z| = 1$ under the linear transformation S in part (a);
- (c) Find out where the exterior region of C is mapped to.
- (d) Find out all linear transformations which map $|z| = 2$ to $|z + 1| = 1$.
- (e) Reflect the line $x = y$ with respect to $|z| = 1$.